EE330 Homework 12 Spring 2024 Solutions

Problem 1

$$I_{D2} = \frac{W_2}{L_2}\frac{L_1}{W_1} I_{D1}$$

$$I_{OUT} = I_{D4} = \frac{W_4}{L_4}\frac{L_3}{W_3} I_{D3}$$

$$I_{D3} = -I_{D2}$$

$$I_{D1} = -250\mu A$$

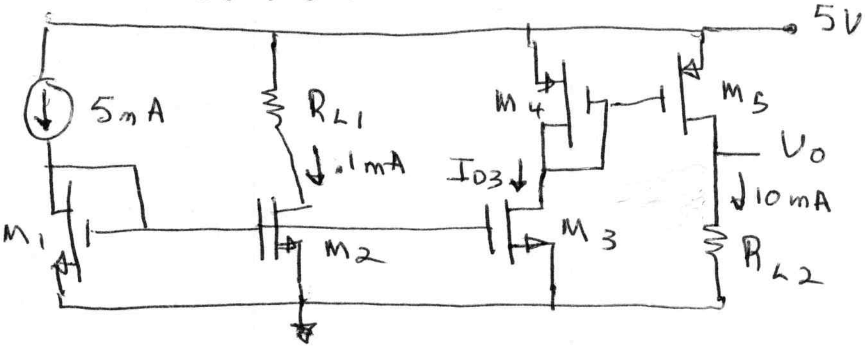
$$\Rightarrow I_{OUT} = \frac{W_4}{L_4}\frac{L_3}{W_3}\frac{W_2}{L_2}\frac{L_1}{W_1} \cdot 250\mu A$$

$$= \left(\frac{5}{1}\right)\left(\frac{6}{20}\right)\left(\frac{4}{2}\right)\left(\frac{1}{5}\right) \cdot 250\mu A$$

$$= 150\mu A$$

Problem 2

a) One solution



$$\left(\frac{W_2}{L_2}\right)\left(\frac{L_1}{W_1}\right)(5mA) = .1 mA$$

Let $W_1 = 50\mu$, $L_1 = 1\mu$

If $L_2 = 1\mu$, $W_2 = 1\mu$

Let $W_3 = W_1$
$L_3 = L_1$

$\therefore I_{D3} = 5mA$

$$\left(\frac{W_5}{L_5}\right)\left(\frac{L_4}{W_4}\right)(5mA) = 10mA$$

Let $W_4 = 40\mu$, $L_4 = 1\mu$

If $L_5 = 1\mu$, $W_5 = 80\mu$

b) To keep M_5 in saturation

$$|V_{DS5}| > |V_{GS5} - V_{TH5}|$$

$$V_{DD} - V_o > |V_{EB5}|$$

but

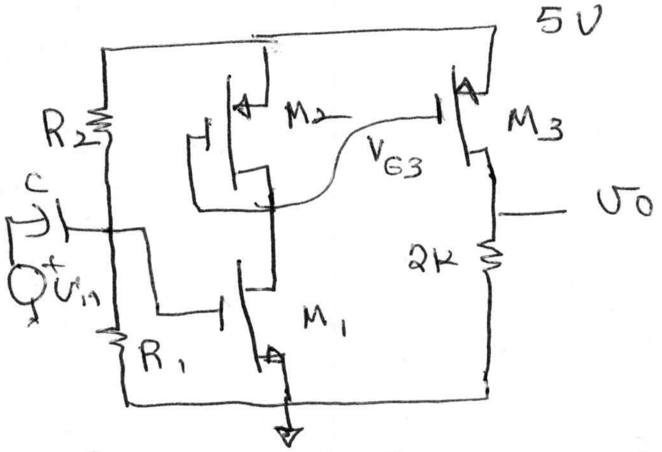
$$I_{D5} = \frac{\mu_p C_{OX} W_5}{2L_5}(V_{GS5} - V_{TH5})^2$$

$$\therefore |V_{GS5} - V_{TH5}| = \sqrt{\frac{10mA}{0.33E-4}\cdot\frac{2}{80}}$$

$$|V_{EB5}| = 2.75V$$

$$\therefore V_o < V_{DD} - |V_{EB5}| = 5 - 2.75 = 2.25V$$

Problem 3 One solution



$$A_v = (-g_{m3})(2K) \times -\frac{g_{m1}}{g_{m2}}$$

$$A_v = (g_{m3})(2K)\left(\frac{g_{m1}}{g_{m2}}\right)$$

Lets set $(g_{m3})(2K) = 1$

$$\frac{g_{m1}}{g_{m2}} = 5$$

Assume C is large

- Consider second stage

$$g_{m3} = \frac{2 I_{DQ3}}{V_{EB3}} \quad \text{so} \quad \frac{2I_{DQ3}}{V_{EB3}} \cdot 2K = 1$$

Set $V_{EB3} = 2V \Rightarrow I_{DQ3} = 0.5mA$, $V_{OQ} = 1V$

$\therefore$ $.5mA = \frac{\mu_p C_{ox} W_3}{2L_3} V_{EB3}^2$ so if $L_3 = 1\mu$, $W_3 = 7.5\mu$

and $V_{G3} = 5V - (V_{EB3} + |V_{TH3}|) = 5 - (2.75V) = 2.25V$

- consider first stage

$$\frac{g_{m1}}{g_{m2}} = 5 \Rightarrow \frac{\mu_n C_{ox} W_1}{L_1} V_{EB1} = \left(\mu_p C_{ox} \frac{W_2}{L_2} V_{EB2}\right) 5$$

but $V_{EB2} = V_{EB3} = 2V$ so $V_{EB1} = V_{EB2} \frac{\mu_p}{\mu_n} \frac{W_2}{L_2} \frac{L_1}{W_1} \cdot 5$

Set $V_{EB1} = 1V \Rightarrow \frac{W_2}{L_2}\frac{L_1}{W_1} = \left(\frac{1}{2}\right)\left(\frac{3}{5}\right) = \frac{3}{10}$

so let $W_1 = 10\mu$, $L_1 = L_2 = 1\mu \Rightarrow W_2 = 3\mu$

- now obtain R_1 & R_2

$$V_{G1} = V_{THn} + V_{EB1} = .75V + 1V = 1.75V$$

$$\left(\frac{R_1}{R_1 + R_2}\right)(5V) = 1.75V.$$

select $R_1 = 100K$

solving find $R_2 = 186K$

check $R_{in} = R_1 // R_2 = 65K$

summary:			
$W_1 = 10\mu$	$W_2 = 3\mu$	$W_3 = 7.5\mu$	$R_1 = 100K$
$L_1 = 1\mu$	$L_2 = 1\mu$	$L_3 = 1\mu$	$R_2 = 186K$

Problem 4 a)

$$\left.\begin{aligned} I_{B1} + I_{B4} &= \frac{10 - 0.6}{R_1} \\ I_{C1} &= \beta I_{B1} \\ I_{C4} &= \beta I_{B4} \end{aligned}\right\} \Rightarrow I_{C1} + I_{C4} = \beta \cdot \frac{9.4V}{R_1} \quad (1)$$

$$\left.\begin{aligned} I_{C1} &= J_S A_{E1} e^{\frac{V_{BE1}}{V_t}} \\ I_{C4} &= J_S A_{E4} e^{\frac{V_{BE4}}{V_t}} \end{aligned}\right\} \Rightarrow$$

since $V_{BE1} = V_{BE4}$

$$I_{C1} = I_{C4} \frac{A_{E1}}{A_{E4}} \quad (2)$$

∴ from (1) and (2)

$$I_{C4} = \frac{\beta(9.4V)}{R_1\left(1 + \frac{A_{E1}}{A_{E4}}\right)}$$

and

$$I_{C1} = \frac{\beta(9.4V)}{R_1\left(1 + \frac{A_{E4}}{A_{E1}}\right)}$$

From the $Q_2 : Q_3$ current mirror

$$V_{o1} = I_{C1} \frac{A_{E3}}{A_{E2}} R_2 = \frac{\beta(9.4V)}{R_1\left(1 + \frac{A_{E4}}{A_{E1}}\right)} \frac{A_{E3}}{A_{E2}} R_2$$

From the $M_5 : M_6$ current mirror

$$V_{o2} = (I_{C4}) \frac{W_6}{L_6} \frac{L_5}{W_5} R_3 = \frac{\beta(9.4V)}{R_1\left(1 + \frac{A_{E1}}{A_{E4}}\right)} \frac{W_6}{L_6} \frac{W_5}{L_5} R_3$$

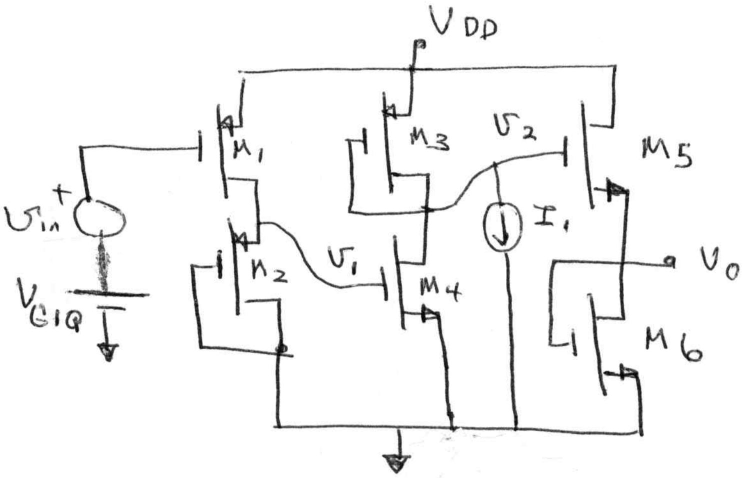
b) From expressions for V_{o1} & V_{o2} from part a)

$$R_2 = 6.1\ K\Omega$$

$$R_3 = 64\ \Omega$$

Problem 5

a)



$$\frac{v_o}{v_{in}} = \frac{v_o}{v_2} \cdot \frac{v_2}{v_1} \cdot \frac{v_1}{v_{in}}$$

$$\frac{v_o}{v_2} = -\frac{g_{m5}}{g_{m6}}$$

$$\frac{v_2}{v_1} = -\frac{g_{m4}}{g_{m3}}$$

$$\frac{v_1}{v_{in}} = -\frac{g_{m1}}{g_{m2}}$$

$$\therefore \quad A_v = -\frac{g_{m1}}{g_{m2}} \cdot \frac{g_{m4}}{g_{m3}} \cdot \frac{g_{m5}}{g_{m6}}$$

b)

$$\frac{g_{m1}}{g_{m2}} = \frac{\frac{2 I_{DQ1}}{V_{EB1}}}{\frac{2 I_{DQ2}}{V_{EB2}}}$$ but $I_{DQ1} = I_{DQ2} \Rightarrow \frac{g_{m1}}{g_{m2}} = \frac{V_{EB2}}{V_{EB1}}$

but $V_{EB1} = 2 - 1 - |V_{TH1}| = .5V$

$V_{EB2} = |V_{GS2}| - |V_{TH2}| = 1 - .5 = .5V$

$$\therefore \frac{g_{m1}}{g_{m2}} = \frac{.5V}{.5V} = 1$$

$$\frac{g_{m4}}{g_{m3}} = \frac{\frac{2 I_{DQ4}}{V_{EB4}}}{\frac{2 I_{DQ3}}{V_{EB3}}}$$ but $I_{DQ3} = I_1 + I_{DQ4} = 2 I_{DQ4}$

so $\frac{g_{m4}}{g_{m3}} = \frac{V_{EB3}}{2 V_{EB4}}$ but $V_{EB3} = V_{DD} - V_{GS} - |V_{Tp}| = 2 - 1.3 - .5 = .2V$

$V_{EB4} = V_{GS4} - V_{Tn} = 1V - .5V = .5V$

$$\therefore \frac{g_{m4}}{g_{m3}} = \frac{.2V}{(2)(.5)V} = 0.2$$

$$\frac{g_{m5}}{g_{m6}} = \frac{\frac{2 I_{DQ5}}{V_{EB5}}}{\frac{2 I_{DQ6}}{V_{EB6}}}$$ but $I_{DQ5} = I_{DQ6} \Rightarrow \frac{g_{m5}}{g_{m6}} = \frac{V_{EB6}}{V_{EB5}}$

but $V_{EB5} = V_{GS5} - V_{Tn} = 1.3 - .7 - .5 = .1V$

$V_{EB6} = V_{GS}$

but $V_{EB5} = V_{GS5} - V_{Tn} = (1.3 - .7) - .5 = 0.1V$

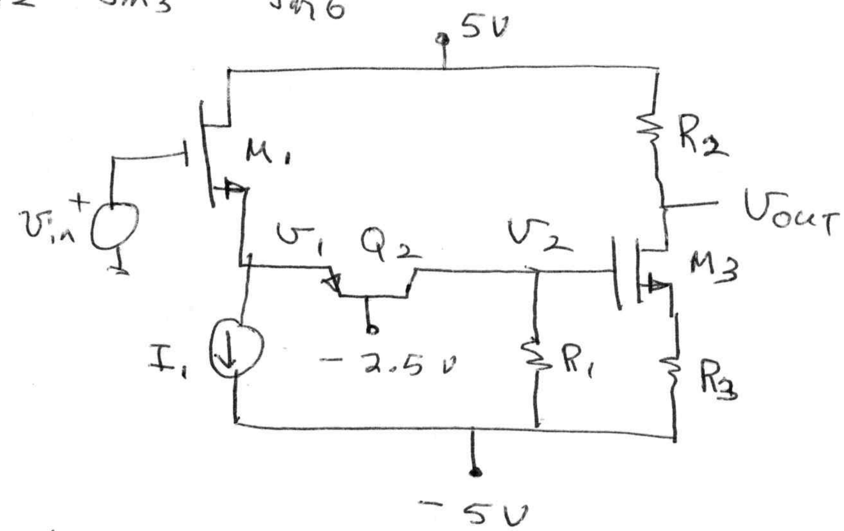
$V_{EB6} = V_{GS6} - V_{Tn} = .7V - .5V = 0.2V$

$\therefore \frac{g_{m5}}{g_{m6}} = \frac{.2V}{.1V} = 2$

It thus follows that

$$A_v = -\frac{g_{m1}}{g_{m2}} \cdot \frac{g_{m4}}{g_{m3}} \cdot \frac{g_{m5}}{g_{m6}} = -1 \cdot (0.2) \cdot (2) = -0.4$$

Problem 6



$$\frac{v_o}{v_{in}} = \frac{v_o}{v_2} \cdot \frac{v_2}{v_1} \cdot \frac{v_1}{v_{in}}$$

$\frac{v_o}{v_2} = -\frac{R_2}{R_3}$ $\qquad \frac{v_2}{v_1} = g_{m2} R_1$ $\qquad \frac{v_1}{v_{in}} \simeq 1$

$$\therefore \frac{v_o}{v_{in}} \simeq -\frac{R_2}{R_3} \cdot g_{m2} R_1$$

Problem 7 $\quad$ Since R_1, R_2 & R_3 given, only need to obtain g_{m2}

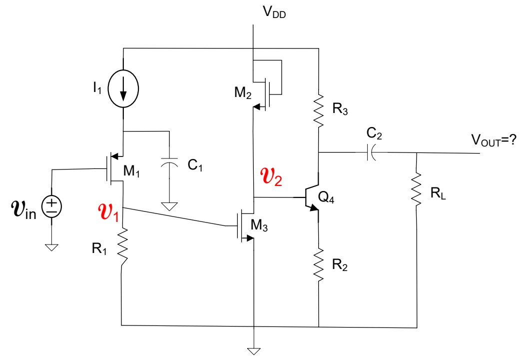
$g_{m2} = \frac{I_{CQ2}}{V_t}$ $\qquad$ but $I_{CQ2} = I_{D1} - I_1$

Observe $V_{S1} \simeq -2.5V + 0.6V = -1.9V$

$\therefore I_{D1} = \frac{\mu_n C_{ox} W_1}{2 L_1} (1.9 - V_{Tn})^2 = 10^{-4} \cdot \frac{8}{4} \cdot (1.4)^2 = .39mA$

$$\therefore \frac{v_o}{v_{in}} = -\left(\frac{20k}{12.5k}\right)\left(\frac{390\mu A - 200\mu A}{25mV}\right)(8k) = -97$$

Problem 8



$$A_V = \frac{v_{OUT}}{v_{IN}} = \frac{v_{OUT}}{v_2} \bullet \frac{v_2}{v_1} \bullet \frac{v_1}{v_{IN}}$$

The last state is a CE with RE stage. The middle stage is a CS stage and the first stage is a CS stage. Thus from the last state

$$\frac{v_{OUT}}{v_2} = -\frac{R_3 // R_L}{R_2}$$

$$R_{IN3} \simeq \beta R_2$$

From the middle stage

$$\frac{v_2}{v_1} = -g_{m3} \bullet \left(\frac{1}{g_{m2}} // R_{IN3} \right)$$

And from the first stage

$$\frac{v_1}{v_{IN}} = -g_{m1} R_1$$

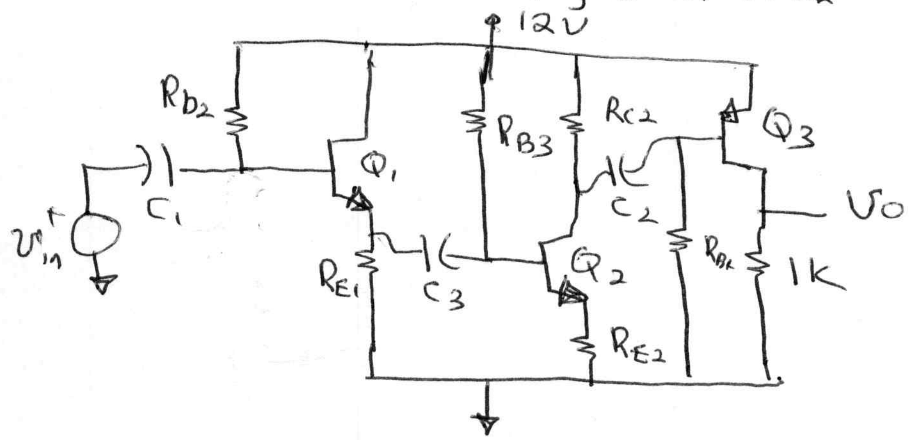
Thus the overall gain is

$$A_V = -\frac{R_3 // R_L}{R_2} - g_{m3} \bullet \left(\frac{1}{g_{m2}} // \beta R_2 \right) \bullet g_{m1} R_1$$

Problem 9 One solution:

3-stage structure. Stage 1: CC to get large input impedance

Stage 2: Inverting Gain Stage   Stage 3: Inverting gain stage



$A_v = 1 \cdot (-g_{m3})(1k) \left(\frac{-R_{C2} \| R_{B4} \| r_{\pi 3}}{R_{E2}} \right)$

$R_{in} = R_{B2} \| \beta [R_{E1} \| \beta R_{E2}]$

Assume C_1, C_2, C_3 large

- Set $g_{m3} \cdot 1k = 60 \Rightarrow \frac{I_{C3}}{V_t} \cdot 1k = 60 \Rightarrow I_{C3} = 1.5mA$

  $\therefore I_{B3} = \frac{I_{C3}}{100} = 15\mu A \Rightarrow \frac{12 - .6}{R_{B4}} = 15\mu A \Rightarrow R_{B4} = 626K$

- Set $(R_{C2} \| R_{B4} \| r_{\pi 3}) / R_{E2} = 1$

  Observe $R_{IN3} = R_{B4} \| r_{\pi 3} = 626K \| \frac{V_t}{I_{B3}} \simeq 1.66K$

  Pick $R_{C2} = 2K \Rightarrow \frac{2K \| 1.66K}{R_{E2}} = 1 \Rightarrow R_{E2} = 907\Omega$

  Pick $I_{C2} = 2mA \Rightarrow V_{B2} = .6 + (2mA)(907\Omega) = 2.4V$

  $\therefore \frac{12 - 2.4}{R_{B3}} = \frac{I_{C2}}{\beta} \Rightarrow R_{B3} = \frac{(9.6)\beta}{2mA} = 480K\Omega$

- Pick $R_{E1} = 5K$, $I_{E1} = 1mA$

  $\therefore V_{B1} = R_{E1} \cdot I_{E1} + .6V = 5.6V$

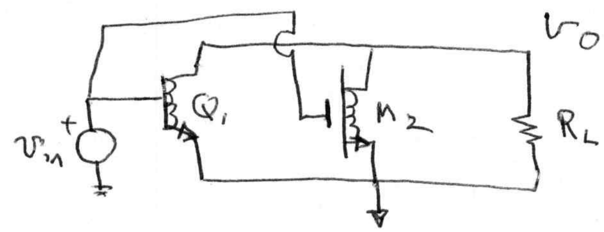
  $I_{B1} = \frac{I_{E1}}{\beta} = \frac{12 - 5.6}{R_{B2}} \Rightarrow R_{B2} = 640K$

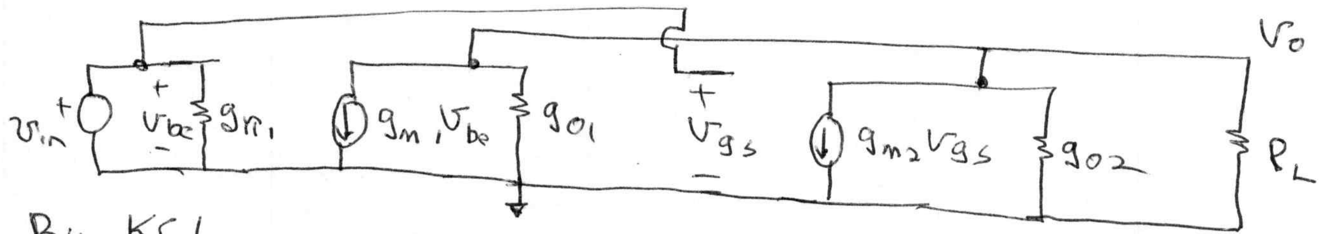
Check $R_{IN} = R_{B2} \| \beta (R_{E1} \| \beta R_{E2}) = 640K \| (100 \cdot [5K \| 90.7K])$

$R_{IN} = 272K\Omega$

Problem 10

a)





By KCL

$$v_o (g_L + g_{o1} + g_{o2}) + g_{m1} v_{in} + g_{m2} v_{in} = 0$$

$$\therefore \frac{v_o}{v_{in}} = -\frac{(g_{m1} + g_{m2})}{g_L + g_{o1} + g_{o2}} \simeq -(g_{m1} + g_{m2}) R_L$$

b) Observe $I_{CQ} = |I_{DQ}|$

$$I_{CQ} = \beta \frac{(0 - (-5 + 0.6))}{R_B} = \beta \cdot \frac{4.4V}{200k} = 2.2mA$$

$$g_{m1} = \frac{I_{CQ}}{V_t} = \frac{2.2mA}{25mV} = 8.8E-2$$

$$g_{m2} = \frac{2|I_{DQ}|}{|V_{EB2}|} = \frac{4.4mA}{1.8 - |V_{THP}|} = \frac{4.4mA}{1.05} = 4.2E-3$$

$$\therefore A_v = -(8.8E-2 + 4.2E-3)(1K) = -92.2$$

so $v_o(t) = (-92.2)(.01 \sin 1000t) = -.92 \sin(1000t)$